UNDERGRADUATE MATHEMATICAL CLUBS.

EDITED BY U. G. MITCHELL, University of Kansas, Lawrence, Kan.

CLUB ACTIVITIES.

MATHEMATICS CLUB OF COLUMBIA UNIVERSITY, New York, N. Y. [*1918*, 227–228].

Officers 1918–19: President, Israel Koral '19; secretary, Charles P. Davis '19; faculty adviser, Professor Lewis P. Siceloff.

Programs of meetings for the current semester are as follows:

February 24, 1919: "Groups of operations" by Professor Cassius J. Keyser.

March 10: "Philosophy of mathematics"[1] by Charles P. Davis '19.

March 24: A visit to the mathematical museum, conducted by Professor James Maclay.

April 7: "Projectiles" by August B. Kinzel '20.

THE MATHEMATICAL CLUB, Harvard University, Cambridge, Mass. [*1918*, 186–7, 449–50].

At the first meeting of the current academic year, held in the Common Room of Conant Hall, February 12, Professor William F. Osgood spoke on "Professor Bôcher's Scientific Start in Life." The following is an extract from the minutes of the meeting.

"Professor Bôcher's student days lay in the years just following the establishment of the principal courses in the Department of Mathematics, as we know them today. Instruction in the Calculus at Harvard, it is true, goes back at least to the middle of the eighteenth century, as is clear from the Commencement programmes, and the first catalogues which print the titles of the courses offered show that formal instruction in the subject was given at least as early as 1830. In the years from 1841 to 1846 Benjamin Peirce published his *Curves and Functions*, which for more than three decades served as a text-book in analytic geometry and the calculus.

"With the coming of Professor Byerly to Harvard in 1876, new and more efficient methods of undergraduate instruction were brought into the Department. The solving of problems by the student as a part of the work of each day's assignment is due to him. He found an ardent and efficient supporter in Professor Benjamin Osgood Peirce, who came into the Department in 1881, and these men gave to Mathematics 2 and 5 [the first and the second course in the Calculus] essentially the character they have today. Both men were interested primarily in the applications of the calculus, and the problems they used bear witness to the fact that they believed the calculus should be taught in its relation to physics. It has been so taught at Harvard ever since.

"Mathematics 4 [Mechanics] was Professor B. O. Peirce's work. On the other hand, Professor Byerly created Mathematics 3 [Modern Geometry]. Advanced courses in geometry had been given in the seventies by Professor James Mills Peirce, who was a pupil of Chasles in Paris in the fifties. But he assigned few problems, and the student found the presentation abstract and difficult to understand. Later, Professor Bôcher and Professor Bouton made useful contributions to the subject matter of the course. But its plan and organization had been completed before Bôcher elected it as a student.

"Two further courses which were to be determinative for Professor Bôcher's scientific work in life were Mathematics 10 [Potential Functions and Developments into Series] and 13 [Theory of Functions of a Complex Variable]. The first of these was created jointly by Professor Byerly and Professor B. O. Peirce, and it took a powerful hold on Bôcher while he was still an undergraduate. Indeed, his thesis for Final Honors in Mathematics—he received Highest Final Honors at graduation—was in this field. It was entitled: 'Three Systems of Parabolic Coördinates.'

[1] Cf. Shaw, J. B., *Lectures on the Philosophy of Mathematics*, Chicago, 1918.

"The other course, Mathematics 13, was given by Mr. (at that time not yet Dr.) Frank Nelson Cole, who had just returned from Germany and was aglow with the enthusiasm which Felix Klein inspired in his students. Cole was not the first to give a formal course of lectures at Harvard on the theory of functions of a complex variable, Professor James Mills Peirce having lectured on this subject in the seventies. That presentation was, however, solely from the Cauchy standpoint, being founded on the treatise of Briot et Bouquet, *Fonctions elliptiques*. Cole brought home with him the geometric treatment which Klein had given in his noted Leipzig lectures of the winter of 1881–82. Cole also gave a course in Modern Higher Algebra, with its applications to geometry. The enthusiasm which he felt for his subject was contagious. Interesting as were the other courses I have mentioned, they stood as the Old over against the New, and of the latter Cole was the apostle. The students felt that he had seen a great light. Nearly all the members of the Department,—Professor J. M. Peirce, Dr. B. O. Peirce, and, I think, Professor Byerly,—attended his lectures. It was the beginning of a new era in graduate instruction in mathematics at Harvard, and mathematics has been taught here in that spirit ever since.

"These were the influences which moulded Bôcher's scientific life when he was yet an undergraduate; this was the atmosphere in which he lived. From here he went to Göttingen, where he studied for six semesters under Klein. An account of his work there will be found in the speaker's forthcoming article on "The Life and Services of Maxime Bôcher," in the *Bulletin of the American Mathematical Society*, May, 1919."

Mathematics Club of the University of Maine, Orono, Me.
[*1918*, 132, 453–454].

During the Fall Term, October to December, 1918, the club was inactive, since the University of Maine was given over to the work of the Student's Army Training Corps, and regular university work was suspended. Many of the members most active in the club last year did not return on account of graduation or war conditions.

In January, 1919, the club was reorganized and the following officers elected. President, Grace H. Hodgdon '19; vice-president, Flavia L. Richardson '20; secretary-treasurer, Edith I. Deering '21; faculty member of the program committee, Professor Myron O. Tripp.

Programs since January, 1919, have been as follows:

January: Social meeting at the home of Dean James N. Hart.

February: "The training of teachers for mathematics" by Professor Myron O. Tripp; "War Savings Stamps as an investment" by Flavia L. Richardson '20.

March: No meeting was held in March.

The Mathematical Club of the University of Nebraska, Lincoln, Nebr.
[*1918*, 313–315].

At the University of Nebraska, as at many other universities, the mathematics club was inactive during the first semester on account of war activities. Soon after the opening of the second semester, the club resumed its activities under the leadership of the officers given below.

Officers, 1919: President, Frances R. Botkin '19; secretary, Mervyn C. Kimberly '20; faculty member of executive committee, Professor Albert Babbitt.

February 13, 1919: "The game of Nim"[1] by Professor Meyer G. Gaba.

[1] A brief discussion of the game of Nim, with references to related literature, was included in Club Topic No. 7, published in the March, 1918, issue of this Monthly (Vol. XXV, pp. 139–142).

March 20: "Complex numbers" by Josiah Brooks '21; "Roots of unity" by Paul J. White '21.

FLATLANDERS—A MATHEMATICAL PLAY IN ONE ACT.[1]

Persons in the Play.

MR. CUBE.
MASTER RATIO—A Schoolmaster, inclined to be fanatical.
CYCLUS—A young nobleman just entering school.
BARON MULTILATUS—Father of Cyclus.

Prologue.

Spoken by Cube.

Dear People of the Third Dimension,
I have to bring to your attention
A place that's hard to understand—
A country that is called *Flatland.*
The people here, as you will see,
Are long or wide—as the case may be,
But one thing they are wont to slight—
They never heard of having *height!*
That woman is a fashion-plate
Whose form's a line that's thin and straight.
And, lest some man should fail to see
A line that's drawn so daintily
And e'er he'd time to step aside
With horrid bump they should collide.
She hums a note that's thin and clear
To let him know she's drawing near.
Since Flatlanders' nobility
Is *sides* instead of ancestry
The King's a circle, and his Prides
Are polygons of many sides.
The Triangle's prestige is small.
The Angle has no name at all.
And yet, unto a Cube like me
The case is grievous as can be.
For, though I'll fight 'gainst circumstance
To pull them *up* from ignorance,
I fear in flight they may not revel—
For Flatlanders are on the *level*!

[1] Performed before The Mathematics Club of Vassar College, February 20, 1919. The Prologue and Epilogue were written by Kathleen Millay '21, and the Scene was adapted by Lucile Free '21. This adaptation of E. A. Abbott's *Flatland* was suggested by a description of a dramatization, in seven scenes, performed at the Haberdashers' Aske's Girls' School, Acton, England, in June, 1913. The description in *School Science and Mathematics*, October, 1914, volume 14, pp. 583–587, was reprinted from *The Mathematical Gazette*, January, 1914, volume 7, pp. 228–231.

The Scene.

Setting: The schoolroom. Around the walls are hung various shaped cardboard figures—inhabitants of Flatland. There is the Teacher's desk, and the benches for the children. In the front row of these are—in line—a cardboard: Triangle; Pentagon; Straight Line; Octagon. Problems treating Areas are on the blackboard. Cyclus is just entering the school. Ratio enters on the other side.

The costumes may be all black and white with cardboard printed names hung around the necks of the characters. Cube may be inside a paper cube if desired.

Time: To-day, in Flatland.

Ratio. Good-Morning, Cyclus. So this is our new pupil.

Cyclus. Good-morning, Master Ratio. Father said you were to teach me my angles very well this year so that I will not make the mistake of associating with people beneath my rank.

Ratio. My students always know their angles very well. But first, let's see how much you know already. You look like a bright lad, and doubtless the son of the great Baron Multilatus has had practice in discovering the rank of his playfellows. Begin over there on the first row, young man, and tell me the class of each child.

Cyclus [*Goes to cardboard figures*]. This, this—let me see—why, this is a Triangle! How stupid of you to admit Triangles to such an exclusive school. I'm sure my father won't approve of it. And this next one—this is a Pentagon.

Ratio. Right, sir.

Cyclus. And this—oh, this is only a woman. I won't even have to bother with it. But this next person seems to trouble me quite a good deal. It couldn't be! Why, yes, you're a Decagon. I would like to walk home from school with you sometime, Mr. Decagon. I'm sure we'll have very much in common. Your father must be a Count.

Ratio. That will do, Cyclus. I see where your trouble lies. By the way, that last gentleman was not a Decagon, but an Octagon. I'm sure we can correct your faults very easily. Now, sir, have you any questions before we begin the lesson for the day?

Cyclus. Please, sir, what is a straight line?

Ratio. A Straight Line is formed by a moving Point.

Cyclus. And what is an Area?

Ratio. An Area is formed by moving a Straight Line.

Cyclus. [*Musing to himself.*] Then I wonder why they couldn't move to the right or left.

Ratio. I beg pardon, sir. You wonder why *who* couldn't move to the right or left?

Cyclus. I was just thinking about a very strange dream I had last night. I was in a strange land. The people all seemed to me to be very unhappy because they were so crowded together. They could not run about and play as we do, but had to move all the time in a Straight Line. They could never pass each other, but when they met all they could do was to go back again the way they had come. It was all so tiresome and stupid. I insulted their king by taking

him for a woman, although how they could tell him from anyone else is more than I can see. A funny Straight Line bumped in to me, and I told him to go to the right. He looked at me in a blank, helpless sort of way, and began to back up. Master Ratio, what do you suppose was the matter?

Ratio. That was a very peculiar dream, Cyclus, but it is all quite clear to me. You see those people only knew one dimension. How much happier they could have been if they had only known that they could move to the right or left in areas. Your dream proves to you, sir, what a lucky person you are. Your life is not hindered by ignorance of the possibilities you have.

Cyclus. [*Hesitating.*] But, Master Ratio, sometimes I think I *am* hindered. I wonder what would happen if you should move an Area.

Ratio. [*Calmly.*] Nothing at all would happen. See, I will illustrate for you. [*He pulls a piece of paper around on the top of his desk.*] There, I have moved it to the right and it is still in the same shape and size. And now to the left—backward,—forward. You cannot change an area. [*There is a knock at the door.*] Come in! [*Cube enters. Master Ratio goes up and feels of or inspects his angles. Of course, all he can see of the cube is a cross section.*] Ah, yes, Mister Square, How do you do? And what can I do for you today?

Cube. I am not a Square; I am a Cube.

Ratio. A what?

Cube. A Cube.

Ratio. And where do you come from?

Cube. I come from the Land-of-the-Third-Dimension.

Ratio. Third Dimension! What a peculiar language you have. According to the meaning of our word *Dimension* there is no third Dimension. And you call yourself a Cube. And yet, you are exactly what we call a Square.

Cube. My language is just the same as yours, Master Ratio, only the range of my vocabulary is much larger. And I tell you again that I am not a Square. If you take a Square and move it up, you will discover what I am.

Ratio. Up?—Up?—Now what can *up* mean? Not this way.—[*Illustrates again pulling his paper over the top of his desk, right, left,* etc.]—nor this, this or this. Cyclus, get me the dictionary, for I must find this word. And what do you mean, Mister Cube, when you say *Third* Dimension?

Cyclus. Maybe we're as ignorant as the people in my dream, after all.

Ratio. [*Irritated.*] No, no, Cyclus! What an absurd idea! What would your father, magistrate of our village, say if he knew you entertained such thoughts! No, no! [*Ratio and Cyclus search dictionaries. Cube inspects figures on the wall. Baron Multilatus enters without knocking.*] What a fruitless search, *Up* is not in any of our dictionaries.

Baron Multilatus. [*Sarcastically.*] Well, well, Master Ratio! I thought I should some day catch you unawares! Of course, *up* is not in the dictionaries. There is no such word. Do you think that you can put any two letters together and make a word? Why not look for *m-i,* or *k-t,* or *p-b, l-l,* anything instead of using *u-p.*

Ratio. But Mister Cube, this fellow here whom we commonly call Mister Square, wants me to move a Square *up* and find the Third Dimension. Baron! [*Dramatically.*] I feel as though I were on the point of making some wonderful new discovery! Something that will help our future generations to live freer, happier lives than ours have been. Something that will ——

Baron Multilatus. Enough of this, my man! I fear you are becoming mentally unbalanced. You'll only end by being burned as a heretic. And if you continue in this fanatical idea of yours I myself shall see that you get your just punishment. Come along, Cyclus—We'll seek a better master, and a saner one.

[*Exeunt Baron Multilatus with Cyclus.*]

Ratio. [*Musing.*] He thinks I'm crazy, does he? Well, well, perhaps I am,—but we'll see. We'll see. [*To Cube.*] And now, friend Cube, I'm off to my fellow-masters to see if I can get any help in this miraculous problem you have suggested. Doubtless they, too, will think me mad. Oh, when will the world learn to respect a man who is seeking after new truths? But—I'll meet you later, sir, and perhaps then we can bring happiness to my poor people. [*Exit Ratio.*]

Epilogue.

Spoken by Cube.

And thus, my friends, this demonstration
Portrays a grievous situation—
A people who would like to find
A way to educate the mind,
And yet, who miss inevitably
A truth we know quite naturally.
And, sometimes, when I contemplate
Their ignorance so desolate
I wonder if *we* fail to see
Some evident reality.
And so, I bring to your attention
The subject of the Fourth Dimension.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about problems to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

2776. Proposed by C. P. SOUSLEY, State College, Penn.

Prove by elementary geometry that the Wallace lines of the extremities of any diameter of the circumscribed circle of a triangle intersect at right angles on the nine-point circle of the triangle.